Statistical Prediction of Ocean Circulation and Trajectories

Greg Holloway
Institute of Ocean Sciences, Sidney, BC V8L 4B2 Canada}
phone: (250)363-6564 fax: (250)363-6746 email: zounds@ios.bc.ca

William Merryfield
Institute of Ocean Sciences, Sidney, BC V8L 4B2 Canada
phone: (250)363-6402 fax: (250)363-6746 email: merryfieldw@dfo-mpo.gc.ca

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LONG-TERM GOALS

We seek to develop a probabilistic description of the evolution of ocean currents and tracer trajectories, in order to improve performance and quantify uncertainty in circulation and forecast models.

OBJECTIVES

In principle, the information content of circulation and forecast models can be expressed in terms of an evolving probability distribution function (pdf) for the flow. We wish to formulate models in terms of moments of the evolving pdf, in particular the mean, which represents a "best guess", and the dispersion, which gauges uncertainty. Such an approach has evident advantages over single-realization models, which provide neither a "best guess" nor a measure of uncertainty, and ensemble models, which sample the pdf but are computationally expensive.

APPROACH

Our approach focuses on entropy as a measure of uncertainty, and the notion that nonlinear dynamical processes drive the pdf up the entropy gradient (toward higher entropy). Currently, we are proceeding on three parallel tracks:

- Analytical exploration of the entropy functional, developing explicit expressions for entropy gradients.

- Simulation of ensembles of quasi-geostrophic flows, relating evolution of moments of the sampled pdf to entropy gradients.

- Extending the determination of maximum-entropy equilibria beyond quasi-geostrophic theory to systems of greater practical significance.

WORK COMPLETED

Codes were developed to compute ensembles of solutions to the *n*-layer quasigeostrophic equations and to the barotropic Ertel potential vorticity equation, with or without topography. An input parameter specifies whether the initial distribution is tightly clustered about some particular state (high initial

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information, low entropy), or is broad and diffuse (low initial information, high entropy). Algorithms were developed for computing accurately the distribution entropy (Carnevale, et al. 1981) and the pair dispersion entropy (Carnevale and Holloway 1982) for cases with and without topography.

To follow up a previous investigation of differential transport of heat and salt by weak stratified turbulence, a code was developed to solve the three-dimensional Boussinesq equations with heat and salt stratification.

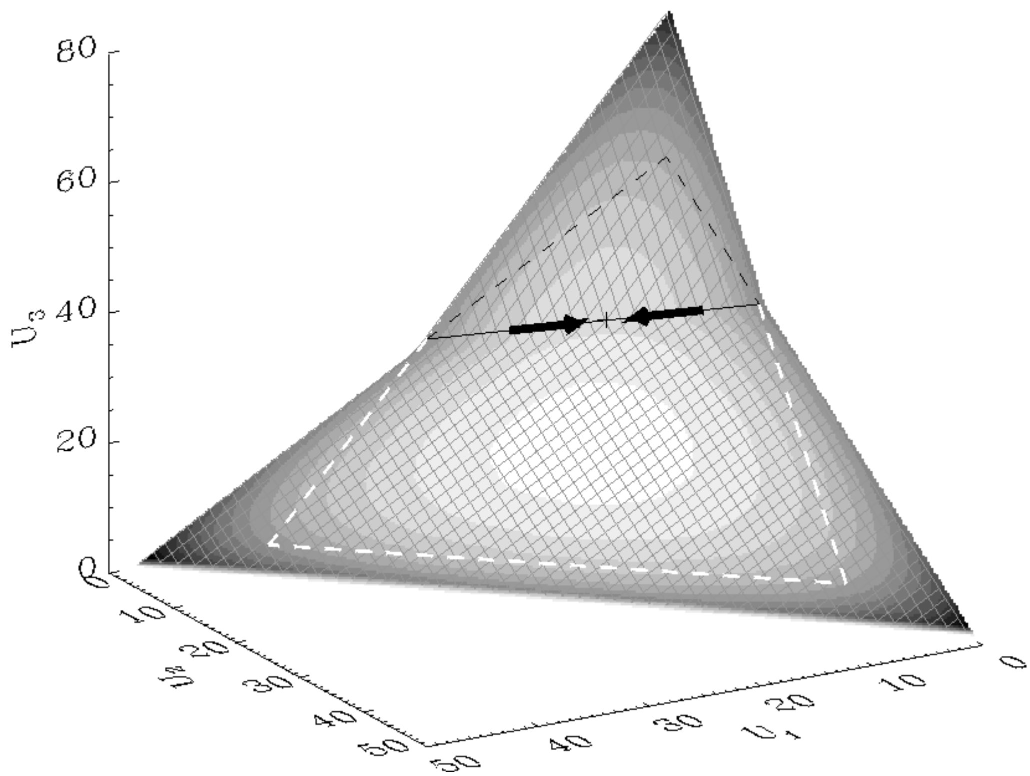
RESULTS

Explicit expressions for entropy gradients were developed as functions of flow pdfs, enabling us to test the hypothesis that entropy gradients force the evolution of pdf moments. To illustrate our approach, consider the simple case of ideal unstratified quasi-geostrophic flow over flat topography, decomposing into N Fourier components. In this instance, entropy is given by $S = \frac{1}{2}\sum_{\mathbf{k}}\ln U_{\mathbf{k}}$ where $U_{\mathbf{k}}$ is ensemble mean energy of the Fourier component having wavenumber $\mathbf{k}$(e.g., Carnevale 1982). Entropy is a function in the N-dimensional phase space, and it is straightforward to find its unconstrained gradient ∇S. However, because energy $E = \frac{1}{2}\sum_{\mathbf{k}}\ln U_{\mathbf{k}}$ and entrosphy $Q = \frac{1}{2}\sum_{\mathbf{k}} k^2 U_{\mathbf{k}}$ are conserved, the system is confined to an N-2 dimensional subspace. The entropy gradient which is relevant is thus ∇S projected onto this subspace.

To illustrate, Fig. 1 shows a low-dimensional analogue having N=3. The two intersecting planes represent the separate dynamical constraints imposed by conservation of E and Q. Light shades indicate high values of S on these surfaces, and dark shades low values. The system must reside somewhere on the N-2 dimensional intersection of the two surfaces (solid line). It is this subspace onto which the N-dimensional entropy gradient must be projected (arrows). The entropy gradient hypothesis states that nonlinearities drive the system in the direction of the arrows.

Entropy gradients calculated in this manner vanish when entropy attains its highest realizable value, indicated by the tick mark in Fig. 1. This approach thus also represents a novel means of determining maximum-entropy equilibria.

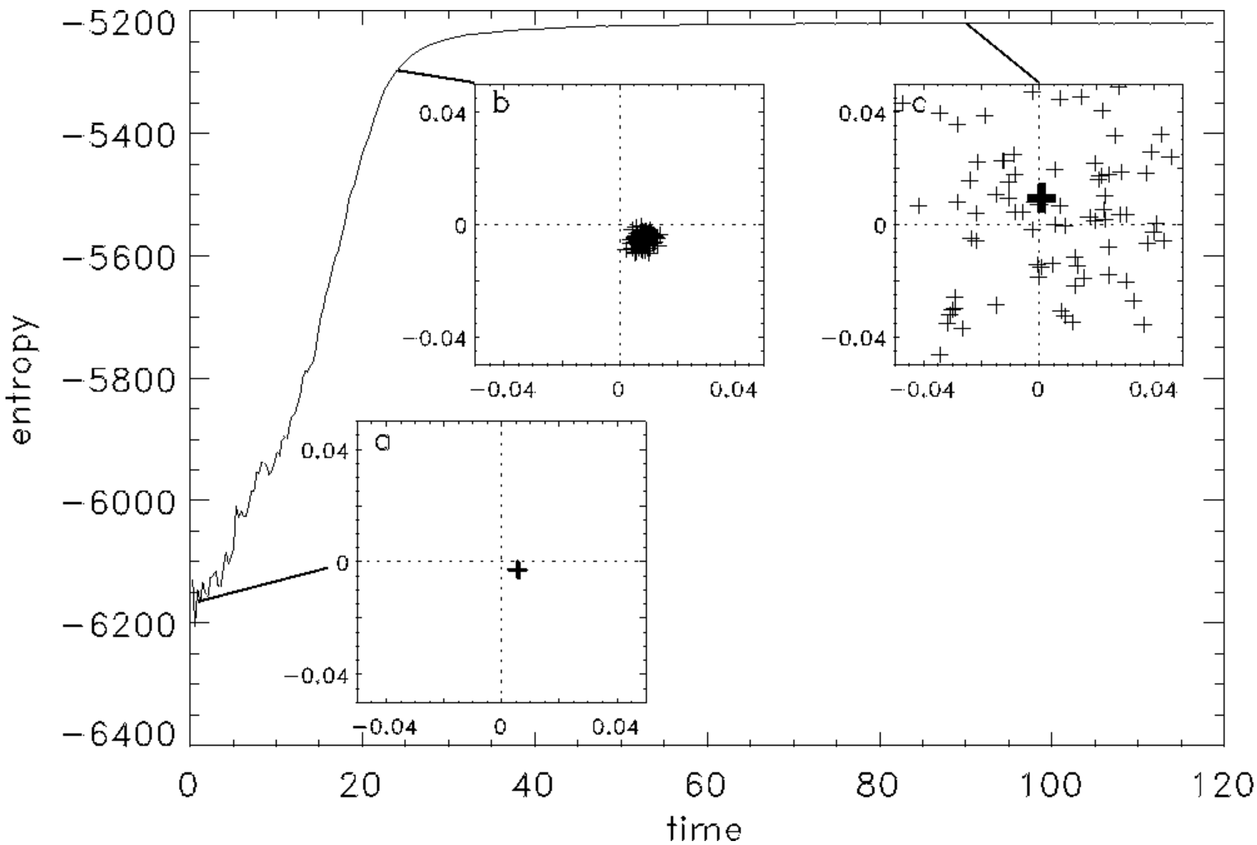
To evaluate the entropy-gradient hypothesis and guide development of equations for the pdf moments, simulations of quasigeostrophic flow ensembles are being conducted. In these calculations, we consider the various realizations in the ensemble as sampling the pdf. An example of how increasing uncertainty translates into increasing entropy for 100 realizations of barotropic flow over flat topography is shown in Fig. 2.



1: Entropy functional for a few-degrees-of-freedom system which conserves energy E and enstrophy Q. Dark shades represent low entropy, and light shades high entropy. Conservation of E and Q constrains the dynamics to the intersection of two planes,indicated by the solid line. (Dashed lines trace invisible portions of the constant-E and constant-Q surfaces.) Arrows indicate entropy gradients for the constrained dynamics, and the tick mark denotes maximum-entropy equilibrium.

In estimating entropy gradients, knowledge of maximum-entropy states is useful because it establishes a basis for computing entropy gradients. Such states have previously been known only for quasigeostrophic flows, which presume infinitesimal topography. This has rendered speculative the application of entropy-gradient concepts to large-amplitude topographies such as coastal margins. To address this, we have evaluated maximum-entropy barotropic flows over finite topography (Merryfield, et al. 1999). Examples for two shelf/slope topographies are shown in the upper panels of Fig. 3. These show that when amplitude of topography is large, mean flows (i) are especially intense, and (ii) peak near the shelf break, rather than over the maximum slope as in the quasigeostrophic case.

The combined influence of stratification and large-amplitude topography was addressed by heuristically combining the results described above with those of Merryfield (1998) for stratified quasigeostrophic flow. The resulting flows reduce to these two cases in the appropriate limits, and satisfy other desirable properties such as divergenceless mass flux (Merryfield, et al. 1999). The lower panels of Fig. 3 show examples of these flows for uniform Brunt-Väisälä frequency $N = 3 \times 10^{-3} s^{-1}$, and illustrate the tendency for bottom trapping which accompanies stratification.



2: Evolution of entropy and the sampled pdf of one (complex) Fourier mode for an ensemble of 100 barotropic quasigeostrophic flows. (a) Initially, pdf dispersion and entropy are low. (b) As ensemble dispersion increases, entropy grows. (c) At large times, dispersion and entropy are large, and the information content of the ensemble is low. (The heavy symbol represents the ensemble mean.)

IMPACT/APPLICATIONS

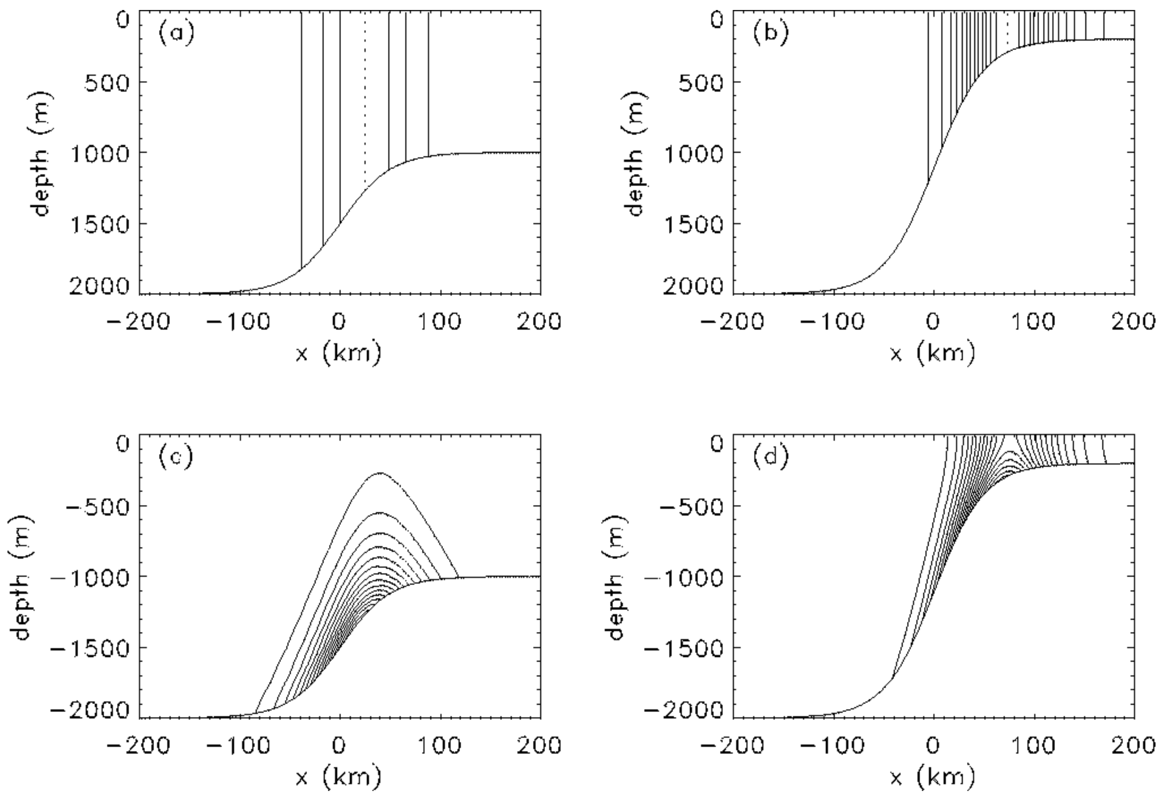
Development of moment-equation based forecast models will improve modeling practice by enhancing the skill of the "best guess" and by quantifying the uncertainty of that "guess" using computationally efficient means.

TRANSITIONS

Our three-dimensional Boussinesq code is being employed by A. Gargett and L. Beauchemin to investigate differential mixing of heat and salt.

RELATED PROJECTS

Holloway leads an Arctic ice/ocean/atmosphere modeling effort as part of an Ocean Climate initiative by the Department of Fisheries and Oceans. Funding to Holloway is 60k/year (Canadian $) for two years with notional commitment to third and fourth years. This work reinforces our ONR-funded research by advancing the state of ocean models used to implement and test subgrid-scale parameterizations.



3: Top: Maximum-entropy barotropic mean flows for two large-amplitude shelf/slope topographies. Bottom: Mean circulations accounting heuristically for stratification according to Merryfield (1998), for uniform Brunt--Väisälä frequency $N = 3 \times 10^{-3} s^{-1}$. Velocities are into the page; contour levels are 1 cm s^{-1} for (a) and (c), 10 cm s^{-1} for (b) and (d).

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